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Parimutuel Applications in Finance

New Markets for New Risks

Ken Baron and Jeffrey Lange

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JEFFREY LANGE

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Ken Baron: *To Debbie*
Jeff Lange: *To my wife Liz*

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List of Symbols

This section defines the symbols that are in the main text of the book. We have excluded notation that is used only in appendices and footnotes.

$\mathbf{A}$	a J by S matrix whose element in the jth row and sth column is $a_{j,s}$ for $j = 1, 2, \ldots, J$ and $s = 1, 2, \ldots, S$;
$a_{j,s}$	a scalar representing the replication weight for customer order j on the sth state claim for $j = 1, 2, \ldots, J$ and $s = 1, 2, \ldots, S$;
a_s	a scalar representing the replication weight on a particular derivative strategy for the sth state claim for $s = 1, 2, \ldots, S$;
b_j	a scalar representing whether customer order j is a buy or a sell order for $j = 1, 2, \ldots, J$;
$\mathbf{c}$	a column vector of length J with element c_j in the jth row for $j = 1, 2, \ldots, J$;
$\mathbf{C}$	a non-empty subset of $\{1, 2, \ldots, J\}$;
c_j	a scalar representing the number of contracts based on the derivative strategy specified in the jth customer order for $j = 1, 2, \ldots, J$;
d	a function of U representing the payout on a derivative strategy;
$\underline{d}$	a scalar representing the minimum payout on a derivative strategy;
$\overline{d}$	a scalar representing the maximum payout on a derivative strategy;
$\mathbf{D}$	a $J + S$ by $J + S$ matrix representing the first derivatives of state prices with respect to customer fills;
d_j	a function of U representing the payout on the derivative strategy requested in customer order j for $j = 1, 2, \ldots, J$;
$\overline{d}_j$	a scalar representing the maximum payout on the derivative strategy requested in customer order j for $j = 1, 2, \ldots, J$;
$\widetilde{d}_s$	a function of U representing the payout on the sth state claim for $s = 1, 2, \ldots, S$;

e a scalar that indexes the E strikes in a parimutuel derivatives auction;
$\tilde{e}$ a scalar that indexes the E strikes in a parimutuel derivatives auction;
E a scalar representing the number of strikes in a parimutuel derivatives auction;
f a function of U representing the P&L on a particular customer order;
F a function of the fills vector $\mathbf{x}$ for the BVIP representation of the GPEP whose jth element is F_j for $j = 1, 2, \ldots, J + S$;
f_j a function of U representing the P&L on the jth customer for $j = 1, 2, \ldots, J$;
F_j a function of the fills vector $\mathbf{x}$ for the BVIP representation of the GPEP for $j = 1, 2, \ldots, J + S$;
$\mathbf{H}$ an S by S matrix relating the PEP to an eigensystem;
j a scalar that indexes the J customer orders in a parimutuel derivatives auction;
J a scalar representing the number of customer orders in a parimutuel derivatives auction;
k a scalar representing an option strike;
k_e a scalar representing the eth strike in a parimutuel derivatives auction for $e = 1, 2, \ldots, E$;
$k_{\tilde{e}}$ a scalar representing the $\tilde{e}$th strike in a parimutuel derivatives auction for $\tilde{e} = 1, 2, \ldots, E$;
M a scalar representing the premium paid in a parimutuel auction;
$\widetilde{M}$ a scalar representing the total amount of market exposure in a parimutuel derivatives auction;
m_s a scalar representing the premium invested in the sth state claim for $s = 1, 2, \ldots, S$;
$m_{\tilde{s}}$ a scalar representing the premium invested in the $\tilde{s}$th state claim for $\tilde{s} = 1, 2, \ldots, S$;
n a scalar that indexes vectors of customer fills;
o_s a scalar representing the odds on the sth horse to win a race for $s = 1, 2, \ldots, S$;
$\mathbf{p}$ a column vector of length S representing the prices of the state claims;
p_s a scalar representing the price of the sth state claim for $s = 1, 2, \ldots, S$;
$p_{\tilde{s}}$ a scalar representing the price of the $\tilde{s}$th state claim for $\tilde{s} = 1, 2, \ldots, S$;
r a scalar representing the requested number of contracts for a customer order;

r_j a scalar representing the requested number of contracts for customer order j for $j = 1, 2, \ldots, J$;
s a scalar that indexes the S states or state claims in a parimutuel auction;
$\tilde{s}$ a scalar that indexes the S states or state claims in a parimutuel auction;
S a scalar representing the number of states or state claims in a parimutuel auction;
SD the standard deviation operator;
s_e a scalar representing the state in which $U = k_e$;
$s_{\tilde{e}}$ a scalar representing the state in which $U = k_{\tilde{e}}$;
t a scalar indexing the observations in a time series;
$^{\mathrm{T}}$ the transpose operator;
u a scalar representing a possible value of U;
U a random variable representing the value of an underlying of interest;
$\widetilde{U}$ a random variable that is a function of U;
u_s a scalar representing a possible value of U if the sth state occurs for $s = 1, 2, \ldots, S$;
u_t a scalar representing the value of U at time t;
V a scalar representing the total limit-order violations of the customer orders;
v_j a scalar representing the amount of the limit-order violation for customer order j for $j = 1, 2, \ldots, J$;
w a scalar representing the limit price for a customer order;
$\mathbf{w}$ a column vector of length J whose element in row j is w_j for $j = 1, 2, \ldots, J$;
w_j a scalar representing the limit price for customer order j for $j = 1, 2, \ldots, J$;
x a scalar representing the number of filled contracts for a customer order;
$\mathbf{x}$ a column vector of customer fills;
X a $(J + S)$-dimensional box representing possible fills for the GPEP;
x_j a scalar representing the number of filled contracts for customer order j for $j = 1, 2, \ldots, J$;
$\mathbf{x}_n$ a column vector of length J of possible customer fills;
$\mathbf{x}_{n,j}$ a scalar representing the customer fill from the nth vector of customer fills for the jth customer order for $j = 1, 2, \ldots, J$;
$y_{n,s}$ a scalar representing the net customer payout from the nth vector of customer fills for the sth state for $s = 1, 2, \ldots, S$;

y_s	a scalar representing the amount of customer payouts required if the sth state occurs for $s = 1, 2, \ldots, S$;
$\mathbf{z}$	a column vector of length $J + S$;
α	a scalar representing a constant strictly between zero and one;
β	a scalar representing a constant;
δ_j	a scalar representing the amount used to adjust the fill on the jth customer order on a particular iteration in part one of the PEP solution algorithm for $j = 1, 2, \ldots, J$;
Δx_j	a scalar representing the change in the fill on the jth customer order between part one and part two of the PEP solution algorithm for $j = 1, 2, \ldots, J$;
ε_t	a scalar representing the error in a time-series model at time t;
κ	a constant that is independent of the state s;
λ_j	a scalar representing the amount of premium paid by customer order j for $j = 1, 2, \ldots, J$;
$\tilde{\lambda}_j$	a scalar representing the amount of market exposure for customer order j for $j = 1, 2, \ldots, J$;
Ω	a set representing the sample space of the random variable U;
π	a scalar representing the price of a derivative strategy;
π^{f}	a scalar representing the price of a forward;
π_j	a scalar representing the price of the derivative strategy requested in customer order j for $j = 1, 2, \ldots, J$;
π^{rf}	a scalar representing the price of a range forward;
ρ	a scalar representing the tick size of U;
θ_s	a scalar representing the amount of opening order premium invested in the sth state claim for $s = 1, 2, \ldots, S$;
ψ_t	a scalar representing the surprise of the underlying at time t;
φ_t	a scalar representing the market's consensus forecast for the value of the underlying at time t;
$\mathbf{0}$	a column vector of length J of all zeros;
$\leftarrow$	an operator that denotes adjusting the variable immediately to the left of the arrow iteratively based on the formula immediately to the right of the arrow.

List of Acronyms

BIS	Bank for International Settlements
BLS	Bureau of Labor Statistics
BVIP	Box Constrained Variational Inequality Problem
CBOT	Chicago Board of Trade
CEA	Commodities Exchange Act
CFMA	Commodity Futures Modernization Act
CFTC	Commodity Futures Trading Commission
CME	Chicago Mercantile Exchange
CPCAM	Convex Parimutuel Call Auction Mechanism
CPI	Consumer Price Index
DARPA	Defense Advanced Research Projects Agency
DPM	Dynamic Parimutuel Market
EIA	Energy Information Administration
FCM	Futures Commission Merchant
GPEP	Generalized Parimutuel Equilibrium Problem
GWS	Gu, Whinston, and Stallaert
LIFFE	London International Financial Futures and Options Exchange
LP	Linear Program
NA	Not Applicable
NFP	Monthly Change in US Nonfarm Payrolls
NYMEX	New York Mercantile Exchange
NYSE	New York Stock Exchange
OTC	Over the Counter
PEP	Parimutuel Equilibrium Problem
P&L	Profit and Loss
SPEP	Simplified Parimutuel Equilibrium Problem
TIPS	United States Treasury Inflation-Protected Securities
UK	United Kingdom
US	United States
USPTO	United States Patent and Technology Office
VIP	Variational Inequality Problem

Foreword

This book is about the technology for managing risks, and about a particular advance in risk management: parimutuel financial markets, the first example of which came into being in 2002 with the Economic Derivatives Market. The market continues today, sponsored by Goldman Sachs, at the service of traders around the world. The authors of this book were responsible for much of the technological innovation that made this historic market possible, and this book explains their methods and reasoning in impressive detail.

The parimutuel technology is fundamentally new in that it stretches the horizons of what can be traded on our risk markets. As such technology is further disseminated, it will mean better management of the widest variety of possible risks, and greater human welfare.

Risk management is important – anyone who has ever collected on an insurance policy will tell you that. But it could be a lot more important. The problem with risk management has been that, while we manage some risks very well, we still do not effectively manage most of our economic risks.

If all risks were really perfectly managed around the world, then, in theory, all idiosyncratic risks would be eliminated, and only shocks that hit the whole world would affect individuals' consumption. Changes in individual consumption in this ideal world of perfect risk management would be perfectly correlated across the nations of the world, since they would only be driven by the irreducible undiversifiable risk that affects everyone. But, in fact the correlation of consumption changes across countries is closer to zero than to one. This means that, for all our sophistication in risk management, we have only just begun the game.

It should be no surprise that most risks are not managed. There simply have been no markets for most of the risks we face. Our principal financial