Springer Series in Solid and Structural Mechanics 2

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Springer

Springer Series in Solid and Structural Mechanics

Volume 2

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ISSN 2195-3511 ISSN 2195-352X (electronic)
ISBN 978-3-319-01801-0 ISBN 978-3-319-01802-7 (eBook)
DOI 10.1007/978-3-319-01802-7
Springer Cham Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013945196

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

In loving memory of
Alf & Signe

Preface

This text book is intended for studies in the theory of structural dynamics, with focus on civil engineering structures that may be described by line-like beam or beam-column type of systems, or by a system of rectangular plates. Throughout this book the mathematical presentation contains a classical analytical description as well as a description in a discrete finite element format, covering the mathematical development from basic assumptions to the final equations ready for practical dynamic response predictions. Solutions are presented in time domain as well as in frequency domain. It has been my intention to start off at a basic level and step by step bring the reader up to a level where the necessary safety considerations to wind or horizontal ground motion induced dynamic design problems can be performed, i.e. to a level where dynamic displacements and corresponding cross sectional forces can actually be calculated. However, this is not a text book in wind or earthquake engineering, and hence, relevant load descriptions are only included in so far as it has been necessary for the performance of illustrative examples. For more comprehensive descriptions of wind and earthquake induced dynamic load and load effects the reader should consult the literature, e.g. refs. [15] and [16]. Less attention has been given to other load cases, e.g. to any kind of shock or impact loading. Also, a comprehensive description of structural damping properties are beyond the scope of this book, but again, for the sake of completeness, a chapter covering the most important theories behind structural damping has been included. The special theory of the tuned mass damper has been given a comprehensive treatment, as this is a theory not fully covered elsewhere. For the same reason a chapter on the problem of moving loads on beams has been included.

The reading of this book will require some knowledge of structural mechanics, i.e. the basic theory of elasticity. Also, readers unfamiliar with the theory of stochastic processes and time domain simulations should commence their studies by reading Appendices A and B, or another suitable text book.

The drawings have been prepared by Anne Gaarden. Thanks to her and all others who have contributed to the writing of this book.

Trondheim, September 2012 *Einar N. Strømmen*

Notation

Matrices and vectors:

Matrices are in general bold upper case Latin or Greek letters, e.g. $\mathbf{K}$ or $\mathbf{\Phi}$.

Vectors are in general bold lower case Latin or Greek letters, e.g. $\mathbf{q}$ or $\boldsymbol{\varphi}$.

$diag[\cdot]$ is a diagonal matrix whose content is written within the bracket.

$det(\cdot)$ is the determinant of the matrix within the bracket.

$tr(\cdot)$ is the trace of a matrix.

Imaginary quantities:

i is the imaginary unit (i.e. $i = \sqrt{-1}$).

$\mathrm{Re}(\cdot)$ is the real part of the variable within the brackets.

$\mathrm{Im}(\cdot)$ is the imaginary part of the variable within the brackets.

Superscripts and bars above symbols:

Super-script T indicates the transposed of a vector or a matrix.

Super-script * indicates the complex conjugate of a quantity.

Dots above symbols (e.g. $\dot{\mathbf{r}}$, $\ddot{\mathbf{r}}$) indicates time derivatives, i.e. d/dt, d^2/dt^2.

Prime on a variable (e.g. C_L' or ϕ') indicates its derivative with respect to a relevant variable, e.g. $\phi' = d\phi/dx$. Two primes is then the second derivative (e.g. $\phi'' = d^2\phi/dx^2$) and so on.

Bar ($-$) above a variable (e.g. $\overline{r}$) indicates its time invariant average value.

Tilde ($\sim$) above a symbol (e.g. $\tilde{M}_n$) indicates a modal quantity.

Hat ($\wedge$) above a symbol (e.g. $\hat{H}_\eta$) indicates a normalised quantity.

The use of indices and superscript:

Index x, y or z refers to the corresponding structural axis.

i and j are general indices on variables.

n and m are mode shape or element numbers.
p and k are in general used as node numbers.

Abbreviations:

CC and SC are short for the centre of cross-sectional neutral axis and the shear centre.

tot is short for total.

max, *min* are short for maximum and minimum.

$\int_L$ or $\int_A$ means integration over the entire length or the area of the system.

Latin letters:

A	Area, cross sectional area
A_j	Coefficient associated with variable j
$A_1^* - A_6^*$	Aerodynamic derivatives associated with the motion in torsion
$\mathbf{A}, \mathbf{A}_m, \mathbf{A}_n$	Connectivity matrix (associated with element m or n)
a	Coefficient, Fourier coefficient, amplitude
$\mathbf{a}_j$	Fourier coefficient vector associated with variable j
B	Cross sectional width
b	Coefficient, band-width parameter
b_c	Distance between cable planes is a suspension bridge
$\mathbf{b}_q$	Buffeting dynamic load coefficient matrix at cross sectional level
C, $\mathbf{C}$	Damping coefficient or matrix containing damping coefficient
C_{ae}, $\mathbf{C}_{ae}$	Aerodynamic damping, aerodynamic damping matrix
c, c_0	Coefficient,damping coefficient at cross sectional level
$\mathbf{c}_0$	Damping matrix at a cross sectional level
$\mathbf{c}, \mathbf{c}_{ae}$	Damping matrix at element level, aerodynamic damping matrix
$Co, \mathbf{Co}$	Co-spectral density, co-spectral density matrix
$\mathbf{Cov}_j$	Covariance matrix associated with variable j
D, d	Cross sectional depth, Coefficient
$\mathbf{d}$, d_k	Element displacement vector, element end displacement component
E	Modulus of elasticity

e , e_c	Exponential number (≈ 2.718281828), Cable sag
$\mathbf{F}$, F	Element force vector, force
f , f_n	Frequency [Hz], eigen frequency associated with mode n
$f(\cdot)$	Function of variable within brackets
G	Modulus of elasticity in shear
$g(\cdot)$, g	Function of variable within brackets, gravity constant
$H(t)$, $\bar{H}$	Horizontal cable force component
$H_1^* - H_6^*$	Aerodynamic derivatives associated with the across-wind motion
H_n , $\mathbf{H}_r$	Frequency response function, frequency response matrix
$\tilde{H}_\eta$, $\tilde{\mathbf{H}}_\eta$	Modal frequency response functions, matrix containing $\tilde{H}_{\eta_n}$
h_c , h_m	Length of suspension bridge hangers, hanger length at mid span
h_r	Vertical distance between shear centre and hanger attachment
h_0	Height (above girder) of suspension bridge tower
I_t , I_w	St Venant torsion and warping constants
I_j	Turbulence intensity of flow components $j = u, v$ or w
I_y , I_z	Moment of inertia with respect to bending about$y$ or z axis
$\mathbf{I}$	Identity matrix
i	The imaginary unit (i.e. $i = \sqrt{-1}$)
J , $\mathbf{J}$	Joint acceptance function, joint acceptance matrix
j	Index variable
K , $\mathbf{K}$	Stiffness, stiffness matrix
K_{ae} , $\mathbf{K}_{ae}$	Aerodynamic stiffness, aero dynamic stiffness matrix
k	Index variable, node or sample number
k_p	Peak factor
$\mathbf{k}$, $\mathbf{k}_{ae}$	Stiffnessmatrix at element level, aerodynamic stiffness matrix
L	Lagrange function
L	Length (of structural system)
${}^{m}L_n$	Integral length scales ($m = y, z$ or θ, $n = u, v$ or w)
ℓ_e	Effective length
M_g , $\mathbf{M}_g$	Concentrated mass at position x_M , mass matrix containing M_g
M_m	Bending moment (m=x, y, z)
m	Index variable
m , $\mathbf{M}$	Mass, mass matrix

$\tilde{m}_n$	Modally equivalent and evenly distributed mass
$\mathbf{m}_0$, $\mathbf{m}$	Mass matrix at a cross sectional level, Mass matrix at element level
N	Number, number of elements in a system
N_r	Number of degrees of freedom in a system
N_x, N_y	Normal force (in x or y directions)
n	Index variable
$\mathbf{n}_n$	Matrix containing time invariant element end forces
P, P_F, P_q	Work performed by external forces acting on the system
$P_1^* - P_6^*$	Aerodynamic derivatives associated with the along-wind motion
p	Index variable, node or sample number
Q_j	External load vector component in directions $j = x, y$ or z
q, $\mathbf{q}$	Pressure, distributed load or load vector at cross sectional level
$R, \mathbf{R}$	External load, reaction force, external load vector at system level
$\tilde{R}, \tilde{\mathbf{R}}$	Modal load, Modal load vector
r, $\mathbf{r}$	Cross sectional displacement or rotation, displacement vector
r_{el}, $\mathbf{r}_{el}$	Element cross sectional displacement, displacement vector
r_p	Polar radius
St	Strouhal number
S, $\mathbf{S}$	Auto or cross spectral density, cross-spectral density matrix
$\mathbf{S}_j$	Cross spectral density matrix associated with variable j
s	General coordinate ($s = x, y$ or z)
T, T_M, T_m	Motion energy of the system body masses
t, T	Time, total length of time window
U, U_M, U_m	Strain energy stored in the material fibres of the system
U	Instantaneous wind velocity in the main flow direction
u	Fluctuating along-wind horizontal velocity component
V	Volume
V, V_R	Mean wind velocity, resonance mean wind velocity
V_y, V_z	Shear forces
v	Fluctuating across wind horizontal velocity component
W_{ext}, W_{int}	External, internal work
w	Fluctuating across wind vertical velocity component
X, Y, Z	Cartesian structural global axis

x, y, z	Cartesian structural element cross sectional main neutral axis (with origo in the shear centre, x in span-wise direction and z vertical)
x_r	Chosen span-wise position for response calculation

Greek letters:

α	Coefficient
β	Phase angle, coefficient
$\boldsymbol{\beta}$	Matrix, matrix containing mode shape derivatives
$\gamma, \gamma_z, \gamma_\theta$	Shear strain, shear strain associated with shear forceor torsion
δ	Incremental displacement operator
∂	Derivative operator
ε , $\boldsymbol{\varepsilon}$, ε_j	Strain, strain vector, strain component ($j = x, y$ or z)
ζ or $\boldsymbol{\zeta}$	Damping ratio or damping ratio matrix
η , $\boldsymbol{\eta}$	Generalised coordinate, vector containing N_{mod} η components
θ	Index indicating cross sectional rotation or load (about shear centre)
κ	Coefficient
ν	Poisson's ratio, coefficient
λ	Coefficient, wave length
μ	Coefficient, friction coefficient
Π	Total energy
ϑ	Coefficient
ρ, ρ_j	Densityof air, density of component associated with j
σ, σ^2	Standard deviation, variance
σ_x , τ	Normal stress, Shear stress
$\phi_{y_n}, \phi_{z_n}, \phi_{\theta_n}$	Continuous mode shape components in y , z and θ directions
$\varphi(x, y)$	Plate mode shape functions
$\boldsymbol{\Phi}$	$3 \cdot N_{\text{mod}}$ by N_{mod} matrix containing all mode shapes $\boldsymbol{\varphi}_n$
$\boldsymbol{\Phi}_r$	3 by N_{mod} matrix containing the content of $\boldsymbol{\Phi}$ at $x = x_r$
$\boldsymbol{\varphi}_n$	Mode shape number n
ψ	Chosen approximate mode shape function, angle
$\boldsymbol{\psi}, \boldsymbol{\psi}_n$	Chosen approximate mode shape matrix, discrete mode shape
$\hat{\boldsymbol{\psi}}, \hat{\hat{\boldsymbol{\psi}}}$	Contains first and second order derivatives of $\boldsymbol{\psi}$

Ω	Coefficient
ω	Circular frequency (rad/s)
ω_n	Eigenfrequency associated with mode shape n
$\omega_n(V)$	Resonance frequency assoc. with mode n at mean wind velocity V

Symbols with both Latin and Greek letters:

$\Delta f, \Delta\omega$	Frequency segment
Δt	Time step
Δs	Spatial separation ($s = x$, y or z)

Contents

Chapter 1
Basic Theory

1.1 Introduction

This text book focuses on the prediction of dynamic response of slender line-like civil engineering structures. It is a general assumption that structural behaviour is linear elastic and that any non-linear part of the relationship between load and structural displacements may be disregarded. It is taken for granted that the load direction throughout the entire span of the structure is perpendicular to the axis in the direction of its span.

It is assumed that the mean value (static part) of any load is constant such that structural response can be predicted as the sum of a mean value and a fluctuating part, as illustrated in Fig. 1.1.a. As shown in Fig. 1.1 and 1.2 a line-like beam or

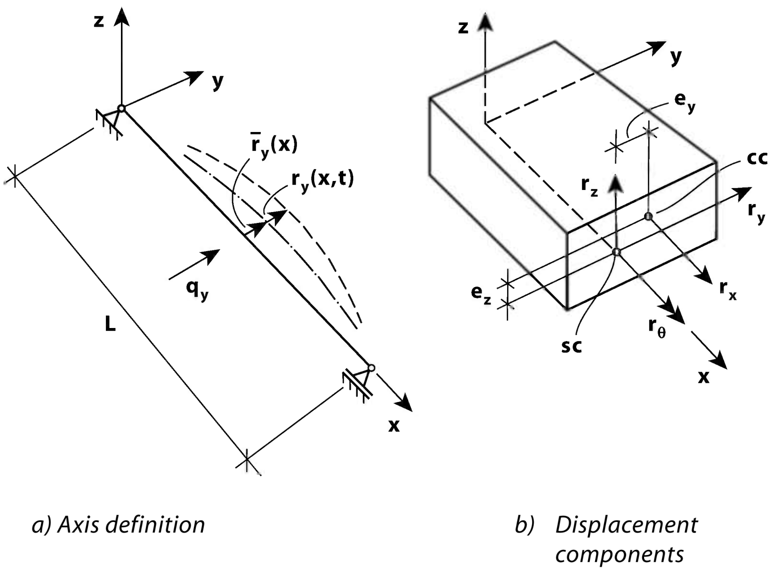

Fig. 1.1 Structural axes and displacement components

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DOI: 10.1007/978-3-319-01802-7_1, © Springer International Publishing Switzerland 2014

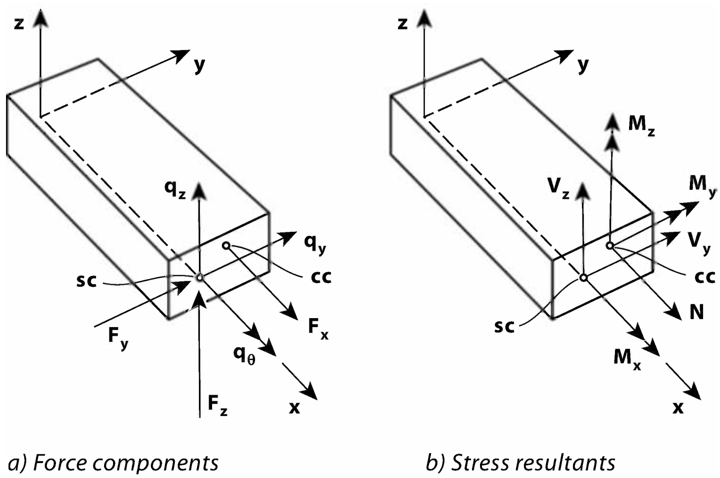

Fig. 1.2 Basic axis and vector definitions

beam-column type of structural element is described in a Cartesian coordinate system $[x, y, z]$, with its origin at the shear centre of the cross section, x is in the span direction and with y and z parallel to the main neutral structural axis CC (i.e. the neutral axis with respect to cross sectional bending according to Hook's law and Navier's hypothesis), which will coincide with the mass centre if material density and modulus of elasticity do not change over the area of the cross section). Response displacements r_y, r_z, r_θ and load components F_y, F_z, q_y, q_z and q_θ are referred to the shear centre (SC), while response displacement r_x and load component F_x are referred to the origin of main neutral axis . Similarly, the cross sectional stress resultants V_y, V_z and M_x are referred to the shear centre, while bending moment and axial stress resultants M_y, M_z and N are referred to the origin of main neutral axis. The basic units are as follows:

- displacement: meter (m)
- time: second (s)
- mass: kilogram (kg)
- force: Newton ($N = kg \cdot m/s^2$, [1, 2]).

1.2 d'Alambert's Principle of Instantaneous Equilibrium

In statics the equilibrium condition of a system subject to a set of constant concentrated forces is given by $\sum F_i = 0$, where F_i (with unit N) are all the relevant forces in the direction of $i = x, y$ or z. It comes from the requirement that a system in static equilibrium must be at rest or in a situation of constant speed, i.e. that its acceleration in any direction $\ddot{r}_i$ (with unit m/s^2) is equal to zero. Newton's second law will then require that $\sum F_i = M \cdot \ddot{r}_i = 0$, where M is the mass of the body. However, in dynamics any equilibrium consideration will have to include the motion of the system. This is done by adopting the principle of d'Alambert (first published by Lagrange [3]) that equilibrium for a system in motion can be established by considering an instantaneous situation where the system is frozen at an arbitrary position in space and time, and that the acceleration of the system can be interpreted as an inertia force in accordance with Newton's second law, i.e. as a resistance against being accelerated.

Discrete Systems

Below, examples of discrete systems are illustrated in Figs. 1.3, 1.5-1.9 and 1.11. For such systems the relevant equilibrium requirements are most conveniently established in a vector-matrix description. Let a system of a simple mass M and a linear elastic spring with stiffness K be suspended in a vertical position as illustrated in Fig. 1.3. To the left the system is shown at its unloaded position. Let the system then be subject to gravity Mg (where g is the gravity acceleration constant) and a constant time invariant force $\overline{F}$. In this position the system is at rest in its static position and it has been displaced a distance $\overline{r}$. As shown in Fig. 1.3.b the equilibrium requirement is then that $K \cdot \overline{r} = M \cdot g + \overline{F}$, from which $\overline{r}$ may be calculated if all other quantities are known. Let the system then be subject to an additional dynamic force $F(t)$, which is accompanied by a corresponding dynamic displacement $r(t)$. The equilibrium condition is then that the external forces $M \cdot g + \overline{F} + F(t)$ must be equal to the sum of the elastic spring force $K \cdot r_{tot}$ and a resistance inertia force $M \cdot \ddot{r}_{tot}$ in accordance with Newton's second law and the principle of d'Alambert, i.e. that

$$M \cdot \ddot{r}_{tot} + K \cdot r_{tot} = M \cdot g + \overline{F} + F(t) \tag{1.1}$$

Introducing that $r_{tot} = \overline{r} + r(t)$ then

$$M \cdot \ddot{r} + K \cdot (\overline{r} + r) = M \cdot g + \overline{F} + F(t) \tag{1.2}$$

Since the static equilibrium condition is that

$$K \cdot \overline{r} = M \cdot g + \overline{F} \tag{1.3}$$

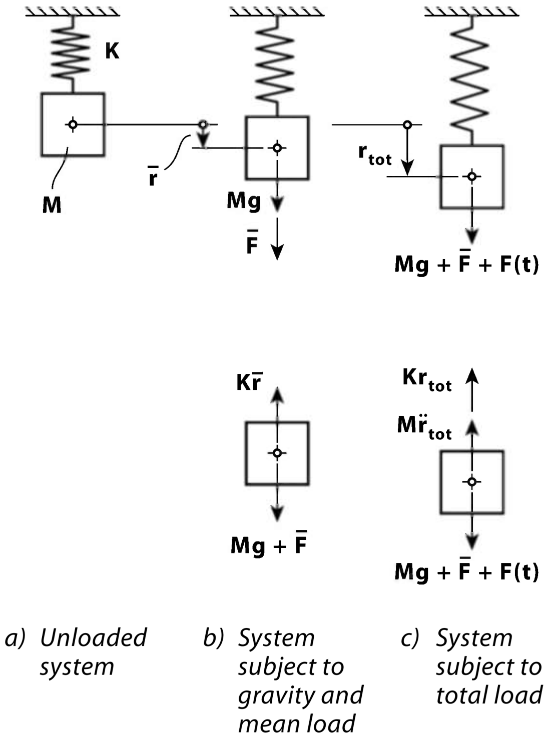

Fig. 1.3 Simple spring-mass system

it is seen that Eq. 1.2 may be reduced accordingly into a purely dynamic equilibrium condition

$$M \cdot \ddot{r} + K \cdot r = F(t) \tag{1.4}$$

Thus, it may be concluded that the equilibrium condition for such a linear elastic system may be split into two, a static time invariant condition and a dynamic equilibrium condition where only dynamic loads are included and where the forces due to the instantaneous acceleration of the system itself is represented by a set of inertia forces acting in the opposite direction of the motion. Hence, by splitting the load (concentrated or evenly distributed) into a mean time invariant part and a fluctuating part

$$\mathbf{F}_{tot} = \bar{\mathbf{F}} + \mathbf{F}(t) = \begin{bmatrix} \bar{F}_x \\ \bar{F}_y \\ \bar{F}_z \end{bmatrix} + \begin{bmatrix} F_x(t) \\ F_y(t) \\ F_z(t) \end{bmatrix} \text{ or } \mathbf{q}_{tot} = \bar{\mathbf{q}} + \mathbf{q} = \begin{bmatrix} \bar{q}_y(x) \\ \bar{q}_z(x) \\ \bar{q}_\theta(x) \end{bmatrix} + \begin{bmatrix} q_y(x,t) \\ q_z(x,t) \\ q_\theta(x,t) \end{bmatrix} \tag{1.5}$$

then the mean and fluctuating parts of the response displacements as well as the corresponding cross sectional stress resultants

$$\bar{\mathbf{r}} + \mathbf{r} = \begin{bmatrix} \bar{r}_y(x) \\ \bar{r}_z(x) \\ \bar{r}_\theta(x) \end{bmatrix} + \begin{bmatrix} r_y(x,t) \\ r_z(x,t) \\ r_\theta(x,t) \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \bar{V}_y(x) \\ \bar{V}_z(x) \\ \bar{M}_x(x) \\ \bar{M}_y(x) \\ \bar{M}_z(x) \\ \bar{N}(x) \end{bmatrix} + \begin{bmatrix} V_y(x,t) \\ V_z(x,t) \\ M_x(x,t) \\ M_y(x,t) \\ M_z(x,t) \\ N(x,t) \end{bmatrix} \tag{1.6}$$

may be obtained by separately satisfying the relevant static and dynamic equilibrium requirements of the system.

Let us first assume that $F(t)=0$, but that the system in Fig. 1.3 has been set into an oscillating motion by imposing an initial displacement $r(0)$ and $\dot{r}(0)$. The solution to Eq. 1.4 is then given by

$$r(t) = b \cdot \sin(\omega_n t) + c \cdot \cos(\omega_n t) \tag{1.7}$$

where b and c are coefficients which may be determined from the position and velocity conditions at $t=0$, i.e. that $r(0)=c$ and $\dot{r}(0)=\omega_n b$, and where the frequency of motion ω_n may be obtained by introducing Eq. 1.7 into Eq. 1.4, from which it is obtained that

$$\left(K - \omega_n^2 M\right) \cdot r(t) = 0 \tag{1.8}$$

A non-trivial solution $r(t) \neq 0$ can then only be obtained if $K - \omega_n^2 M = 0$, and thus, the frequency of a free unloaded and oscillatory motion is given by $\omega_n = \sqrt{K/M}$. The motion is harmonic because it contains only a single and stationary frequency. This is what we call the eigenfrequency of the system. It has the unit rad/s. [In some cases it may be convenient to convert it into $f_n = \omega_n/(2\pi)$ Hz (1/s), while yet another option is to introduce the period of the motion $T_n = 1/f_n$.] Furthermore, since for two arbitrary angles α_1 and α_2

$$\sin\alpha_1 \cdot \sin\alpha_2 + \cos\alpha_1 \cdot \cos\alpha_2 = \cos(\alpha_1 - \alpha_2) \tag{1.9}$$