Trends in Mathematics

José Luis Cisneros-Molina
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Editors

Singularities in Geometry, Topology, Foliations and Dynamics

A Celebration of the 60th Birthday of José Seade, Merida, Mexico, December 2014

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Trends in Mathematics

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Contents

Preface

The workshop “Singularities in geometry, topology, foliations and dynamics” took place in Mérida, Mexico, from December 8 to 19, 2014. It was a celebration of José Seade’s 60th birthday. This meeting was preceded by a two week long school, held at the Institute of Mathematics of Universidad Nacional Autónoma de México (UNAM), in Cuernavaca, Mexico.

The workshop was held in a historical building of the Universidad Autónoma de Yucatán, located in downtown in Mérida. It was supported and financed by various entities of UNAM (Instituto de Matemáticas, Posgrado de Matemáticas, Dirección General de Asuntos del Personal Académico), as well as by the Consejo Nacional de Ciencia y Tecnología (CONACyT) and the Abdus Salam International Centre for Theoretical Physics (ICTP). The main organizing institution was the Institute of Mathematics of UNAM.

During the two weeks of the workshop, a total of forty-four plenary talks were presented, as well as ten poster presentations. There were a total of 121 participants coming from 14 different countries, a list of which appears below. The themes in singularity theory discussed at this meeting include the topology of singularities and characteristic classes, resolutions of spaces and of foliations, contact structures, Milnor fibrations, metric and bi-Lipschitz behaviour, equisingularity, moduli of spaces and foliations, among others.

José Seade, also known to his colleagues as Pepe Seade, was originally trained as an algebraic topologist at the University of Oxford, where he wrote his Ph.D. thesis under the direction of Brian Steer and Nigel Hitchin. Since his very first publications he showed his interest into singularities. Over a period of 35 years of productive research, Pepe’s work in singularity theory has explored a variety of subthemes: vector fields, characteristic classes, mappings and foliations, Milnor fibrations, contact structures, and the topology of local singularities. Even then, his strong dedication to the field of singularities has not prevented him from working in other fields, such as Kleinian groups and dynamical systems, where his research has also had an unmistakable impact. Since 1981, Pepe has published 63 research papers as well as 4 books. Two of his books have been awarded the Ferran Sunyer i Balaguer prize: one a book on the topology of singularities and the other a book on complex Kleinian groups.

Pepe has also played an important role in integrating the Mexican mathematical community into a variety of important international mathematical networks. This is due in large part to his abilities in organizing international meetings and facilitating the formation of research groups, as well as his readiness to help young people obtain financial support or make scientific contacts abroad. These activities – in Mexico, America, and worldwide – have helped make Mexico an international center for singularity theory.

These are just some of the reasons explaining why so many mathematicians from all over the world attended the workshop.

This volume consists of 13 original research articles – submitted by some of the participants of the workshop – covering various aspects of singularity theory. At least one co-author of each paper was present at the conference or took part in its preparation.

The scientific committee of the workshop consisted of Roberto Callejas-Bedregal (Universidade Federal da Paraíba, Brazil), José Luis Cisneros-Molina (UNAM, Cuernavaca, Mexico), Javier Fernández de Bobadilla (Instituto de Ciencias Matemáticas, Spain; Institute for Advanced Study, Princeton, USA), Xavier Gomez-Mont (CIMAT, Mexico), Renato Iturriaga (CIMAT, Mexico), Anatoly Libgober (University of Illinois at Chicago, USA), David Massey (Northeastern University, USA), Mutsuo Oka (Tokyo University of Science, Japan), Anne Pichon (Institut de Mathématiques de Luminy, France), Marcelo Saia (ICMC, USP, São Carlos, Brazil), Jawad Snoussi (UNAM, Cuernavaca, Mexico), Mark Spivakovsky (Institut de Mathématiques de Toulouse, France), Alberto Verjovsky (UNAM, Cuernavaca, Mexico).

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The members of the editorial committee are grateful to all the referees who did a fantastic job in reviewing all the submitted papers, sometimes proposing interesting and useful modifications. We would like also to thank the entire team of the Trends in Mathematics series for their wonderful work.

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Extending the Action of Schottky Groups on the Complex Anti-de Sitter Space to the Projective Space

Vanessa Alderete, Carlos Cabrera, Angel Cano and Mayra Méndez

This paper is dedicated to Pepe Seade in celebration of his 60th Birthday Anniversary.

Abstract. In this article we show that if a complex Schottky group, acting on the complex anti-de Sitter space, acts on the corresponding projective space as a Schottky group, then the space has signature (k, k). As a consequence, we are able to show the existence of complex Schottky groups, acting on $\mathbb{P}^n_{\mathbb{C}}$, such that the complement of whose Kulkarni's limit set is not the largest open set on which the group acts properly and discontinuously. This is the starting point towards the understanding of the notion of the role of limit sets in the higher-dimensional setting.

Mathematics Subject Classification (2000). Primary 37F30, 32M05, 32M15; Secondary 30F40, 20H10, 57M60.

Keywords. Schottky groups in higher dimensions, limit sets, complex hyperbolic spaces.

Introduction

Classical Schottky groups in $\mathrm{PSL}(2, \mathbb{C})$ play a key role in both complex geometry and holomorphic dynamics. On one hand, Koebe's Retrosection Theorem says that every compact Riemann surface can be obtained as the quotient of an open set in the Riemann sphere which is invariant under the action of a Schottky group. On the other hand, the limit sets of Schottky groups have a rich and fascinating geometry and dynamics, which has inspired much of the current knowledge we have about fractal sets and 1-dimensional holomorphic dynamics. In this article

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we study the behavior of complex Schottky groups in $\mathrm{PU}(k,l)$ acting on $\mathbb{P}_{\mathbb{C}}^{k+l-1}$. More precisely we show:

Theorem 0.1. *If a purely loxodromic free discrete subgroup of* $\mathrm{PU}(k,l)$ *acts as a complex Schottky group on* $\mathbb{P}_{\mathbb{C}}^{k+l-1}$*, then* $k=l$*. Moreover in this case,*

1. *the group* Γ *acts as a complex Schottky group on the complex anti-de Sitter space;*
2. *the limit set* $\Lambda_{PA}(\Gamma)$ *is contained in the complex anti-de Sitter space and homeomorphic to the product* $\mathcal{C}\times\mathbb{P}_{\mathbb{C}}^{k-1}$*, where* $\mathcal{C}$ *is the triadic Cantor set.*

The limit set $\Lambda_{PA}(\Gamma)$ will be defined in Theorem 1.9. As a partial reciprocal of the previous theorem we have:

Theorem 0.2. *Let* $\Gamma\subset\mathrm{PU}(k,k)$ *be a group acting as a complex Schottky group on the complex anti-de Sitter space. If* Γ *is generated by* $\gamma_1,\dots,\gamma_n$*, then there is* $N\in\mathbb{N}$ *such that* $\Gamma_N=\langle\langle\gamma_1^N,\dots,\gamma_n^N\rangle\rangle$ *acts as a complex Schottky group on* $\mathbb{P}_{\mathbb{C}}^{2k-1}$.

The paper is organized as follows: in Section 1, we review some general facts and introduce the notation used along the text. In Section 2, we answer a question by J. Parker showing that no complex Schottky group is hyperbolic. In Section 3, we provide a lemma that helps us to describe the dynamics of compact sets. Finally, in Section 4, we provide a proof of the main results of this article.

1. Preliminaries

1.1. Projective Geometry

The complex projective space $\mathbb{P}_{\mathbb{C}}^n$ is defined as:

$$\mathbb{P}_{\mathbb{C}}^n=(\mathbb{C}^{n+1}\setminus\{0\})/\mathbb{C}^*\,,$$

where $\mathbb{C}^*$ acts by the usual scalar multiplication. This is a compact connected complex n-dimensional manifold equipped with the Fubini-Study metric d_n.

If $[\,]:\mathbb{C}^{n+1}\setminus\{0\}\to\mathbb{P}_{\mathbb{C}}^n$ is the quotient map, then a non-empty set $H\subset\mathbb{P}_{\mathbb{C}}^n$ is said to be a projective subspace of dimension k if there is a $\mathbb{C}$-linear subspace $\widetilde{H}$ of dimension $k+1$ such that $[\widetilde{H}\setminus\{0\}]=H$. In this article, $e_1,\dots,e_{n+1}$ will denote the standard basis for $\mathbb{C}^{n+1}$.

Given a set of points S in $\mathbb{P}_{\mathbb{C}}^n$, we define:

$$\mathrm{Span}(S)=\bigcap\{P\subset\mathbb{P}_{\mathbb{C}}^n\mid P\text{ is a projective subspace containing }S\}.$$

Clearly, $\mathrm{Span}(S)$ is a projective subspace of $\mathbb{P}_{\mathbb{C}}^n$.

1.2. Projective and Pseudo-projective Transformations

Every linear isomorphism of $\mathbb{C}^{n+1}$ defines a holomorphic automorphism of $\mathbb{P}_{\mathbb{C}}^n$. Also, it is well known that every holomorphic automorphism of $\mathbb{P}_{\mathbb{C}}^n$ arises in this way. The group of projective automorphisms of $\mathbb{P}_{\mathbb{C}}^n$ is defined by:

$$\mathrm{PSL}(n+1,\mathbb{C}):=\mathrm{SL}(n+1,\mathbb{C})/\mathbb{C}^*,$$

where $\mathbb{C}^*$ acts by the usual scalar multiplication. Then $\mathrm{PSL}(n+1,\mathbb{C})$ is a Lie group whose elements are called projective transformations. We denote by $[[\ ]]: \mathrm{SL}(n+1,\mathbb{C}) \to \mathrm{PSL}(n+1,\mathbb{C})$ the quotient map. Given $\gamma \in \mathrm{PSL}(n+1,\mathbb{C})$, we say that $\widetilde{\gamma} \in \mathrm{SL}(n+1,\mathbb{C})$ is a lift of γ if $[[\widetilde{\gamma}]] = \gamma$.

1.3. Complex Anti-de Sitter Space and its Isometries

Let us start by constructing the complex anti-de Sitter space. To do that consider the following Hermitian matrix:

$$H = \begin{pmatrix} & & & & & & 1 \\ & & & & & \iddots & \\ & & & & 1 & & \\ & & & \mathrm{Id}_{l-k} & & & \\ & & 1 & & & & \\ & \iddots & & & & & \\ 1 & & & & & & \end{pmatrix}$$

where Id_{l-k} denotes the identity matrix of size $(l-k)\times(l-k)$, and the off-diagonal blocks in the upper right and the lower left are of size $k \times k$. We set

$$U(k,l) = \{g \in \mathrm{GL}(k+l,\mathbb{C}) : gHg^* = H\}$$

and denote by $\langle,\rangle : \mathbb{C}^{n+1} \to \mathbb{C}$ the Hermitian form induced by H. Clearly, $\langle,\rangle$ has signature (k,l), and $U(k,l)$ is the group preserving $\langle,\rangle$, see [11]. The respective projectivization $\mathrm{PU}(k,l)$ preserves the set

$$\mathbb{H}^{k,l}_{\mathbb{C}} = \{[w] \in \mathbb{P}^n_{\mathbb{C}} \mid \langle w,w\rangle < 0\},$$

which is the pseudo-unitary complex ball. We call the boundary, denoted by $\partial\mathbb{H}^{k,l}_{\mathbb{C}}$, *the complex anti-de Sitter space*. In the rest of the article we will be interested in studying those subgroups of $\mathrm{PSL}(n+1,\mathbb{C})$ preserving the pseudo-unitary complex ball.

Given a projective subspace $P \subset \mathbb{P}^n_{\mathbb{C}}$ we define

$$P^{\perp} = [\{w \in \mathbb{C}^{n+1} \mid \langle w,v\rangle = 0 \text{ for all } [v] \in P\} \setminus \{0\}].$$

An important tool in this work is the following result, see [6,11].

Theorem 1.1 (Cartan Decomposition). *For every $\gamma \in \mathrm{PU}(k,l)$ there are elements $k_1,k_2 \in \mathrm{PU}(n+1)\cap\mathrm{PU}(k,l)$ and a unique $\mu(\gamma) \in \mathrm{PU}(k,l)$, such that $\gamma = k_1\mu(\gamma)k_2$ and $\mu(\gamma)$ have a lift in $\mathrm{SL}(n+1,\mathbb{C})$ given by*